Vassil Sgurev

An essay on complex valued propositional logic

Abstract. In the decision making logic it is often necessary solving of logical equations for which, due to the features of disjunction and conjunction, no admissible solutions exist. An approach is suggested in which by introducing of Imaginary Logical Variables (ILV) the classical propositional logic is extended to a complex one. This provides a possibility to solve a large class of logical equations.
The real and imaginary variables each satisfy the axioms of the Boolean algebra and of the lattice. It is shown that the Complex Logical Variables (CLV) observe the requirements of the Boolean algebra and the lattice axioms. Suitable definitions are found for these variables for the operations disjunction, conjunction, and negation.
A series of results are obtained, included also the truth tables of the operations disjunction, conjunction, negation, implication, and equivalence for complex variables. Inference rules are deduced for them analogous to Modus Ponens and Modus Tollens in the classical propositional logic.
Values of the complex variables are obtained, corresponding to TRUE (T) and FALSE (F) in the classic propositional logic. A conclusion may be made from the initial assumptions and the results attained, that the imaginary logical variable i introduced hereby is 'truer' than the condition 'T' of the classic propositional logic and ¬i – 'falser' than the condition 'F', respectively.
Possibilities for further investigations of this class of complex logical structures are pointed out.

Keywords: propositional logic, logical equations, complex propositional logic, Boolean algebra, imaginary logical variable, lattice.

Introduction

Various types of logical equations are beginning to play more and more important role in the last decade in the development and application of different decision support systems. In some of such equations where operations disjunction and conjunction are applied there are no admissible solutions, which fact seriously hinders their usage. Propositional logic is well examined [1, 2] and it has a developed, up-to-date, apparatus to be applied to various areas of knowledge. A good example of this is the decision making logic [3] which is also based on its principles. At last time solving of different classes of logical equations is necessary in different application areas.

An approach is proposed in the present work, which provides a possibility to evade the existing difficulties Due to the specificity of defining the logical operation disjunction and conjunction very often a solution of these equations cannot be found.

As an example the following equation of the propositional logic may be pointed out:

$$\mathrm{F} \wedge X = \mathrm{T}; \tag{1}$$

where X is a logical value which accepts one of two states – True (T) or False (F).

It is evident that in the frame of the classical propositional logic there is no such value of X, $X \in \{T, F\}$. for which the requirements of requirements of equation (1) to be satisfied.

An analogy to this condition may be sought in the number theory in which exist equations of the type:

$$x^2 = -1; x = \sqrt{-1} \tag{2}$$

Equation (2) has no solution in the frame of the real numbers theory, as no real number exists which raised by square to give result -1. This leads to extending the range of the real numbers and to transition to complex ones through the introduction of the imaginary unit i ($i^2 = -1$). The general appearance of the complex number z is:

$$z = a+bi; \tag{3}$$

where a and b are real numbers.

The complex numbers theory turned to be an exclusively fruitful scientific abstraction whose results led to successful solution of a series of problems in the area of science, engineering and economy.

1. Imaginary logical variables

Imaginary valued variables are considered in [9], but in an essentially different way, and [10] and [11] concern the classical Aristotelian logic.

It is expedient the same approach to be used, and solution of equation (1) to be found by introducing an imaginary logical variable in the following way:

$$F \wedge i \equiv T; \quad (4)$$

where "≡" stands as usually, for 'by definition'.

State i and its negation $\neg i$ are a part of the set of the possible imaginary states

$$I = \{i, \neg i\}. \quad (5)$$

The Imaginary Logical Variable (ILV) p may be in one of the two states

$$p \in \{i, \neg i\}. \quad (6)$$

In [5] authors suggest imaginary logical variables, but in the so called quaternion logic, which is substantially from the present approach.

The classical logical variable

$$x \in X=\{T, F\} \quad (7)$$

will be further referred to as Real Logical Variable (RLV).

On the base of relation (4) and by the logical operations disjunction (V), conjunction (Λ) and negation (¬) the complex logical variable (CLV) will be introduced, which is of the type:

$g_1 = x_1 \vee p_1$; $g_2 = x_2 \wedge p_2$; (8)

where x_1 and x_2 are RLV or their negations, and p_1 and p_2 – ILV or their negations. The set of all possible complex logical variables will be denoted by

$$G = \{ g_1; g_2; \dots g_i \dots\}. \quad (9)$$

Real logical variables of the propositional logic sequentially correspond to the algebraic structures symmetric idempotent semi ring, Boolean algebra and a lattice [4].

The Boolean algebra $B_1 = (B'_1, \vee, \wedge, \neg, 0, 1)$ is a structure of two binary operations V and Λ, negation ¬, identity elements 1 and 0 respectively. It is a symmetric idempotent ring in which for each element x a complement $\neg x$ exists, such that

$$x \vee \neg x = 1;\ x \wedge \neg x = 0. \quad (10)$$

The Boolean algebra axioms may be written down in Table 1 as follows:

Table 1

№	Axiom	№	Axiom
1.	$a \vee (b \vee c) \equiv (a \vee b) \vee c$	8.	$a \wedge (b \wedge c) \equiv (a \wedge b) \wedge c$
2.	$a \vee b \equiv b \vee a$	9.	$a \wedge b \equiv b \wedge a$
3.	$a \vee a \equiv a$	10.	$a \wedge a \equiv a$
4.	$a \vee 0 \equiv a$	11.	$a \wedge 1 \equiv a$
5.	$a \wedge (b \vee c) \equiv (a \wedge b) \vee (a \wedge c)$	12.	$a \vee (b \wedge c) \equiv (a \vee b) \wedge (a \vee c)$
6.	$a \wedge 0 \equiv 0$	13.	$a \vee 1 \equiv 1$
7.	$a \vee \neg a \equiv 1$	14.	$a \wedge \neg a \equiv 0$

Three important properties follow from axioms above:

a) The complement operation is symmetric, i.e.

$$\neg\neg a = a; \quad (11)$$

b) For each a the complement $\neg a$ is unique;

c) The following rules, called de Morgan's laws, exist:

$$\neg(a \vee b) = \neg a \wedge \neg b;\quad \neg(a \wedge b) = \neg a \vee \neg b. \quad (12)$$

If the property 'absorption'

$$a \vee (a \wedge b) = a,\ a \wedge (a \vee b) = a, \tag{13}$$

is added to the above stated axioms, then we reach to the algebraic structure lattice.

There are no difficulties of principle the imaginary logical variables to satisfy, like the real ones, the axioms from Table 1 and relations (10) to (13), i.e., the requirements of the symmetric idempotent semi ring are satisfied in them also, as well as the requirements of the Boolean algebra $B_2 = (B'_2, \text{V}, \Lambda, \neg, 0, 1)$, and of lattice. At that as a difference from the RLV variable, the parameters *0* and *1* (identities) will be distinguished from those in B_1.

Hence the RLV in the frame of the Boolean algebra B_1 and the ILV – in the Boolean algebra B_2 function independently from each other in the corresponding algebraic structures. At that always $x \in \{\text{T}, \text{F}\}$, and $p \in \{\text{i}, \neg\text{i}\}$.

2. Connecting RLV and ILV

The complex logical variables may be considered as connecting member between RLV and ILV. In the case that we consider this link is equation (1).

It follows that the way should be defined, in which the logical operations with complex logical variables are performed and to define the frames of the algebraic structures in which they function. It is expedient to define the operations disjunction and conjunction between complex logical values in the following way:

$$(x_1 \vee p_1) \vee (x_2 \vee p_2) = (x_1 \vee x_2) \vee (p_1 \vee p_2)\,; \tag{14}$$

$$(x_1 \wedge p_1) \wedge (x_2 \wedge p_2) = (x_1 \wedge x_2) \wedge (p_1 \wedge p_2); \tag{15}$$

$$(x_1 \vee p_1) \wedge x_2 = (x_1 \wedge x_2) \vee (x_2 \wedge p_1); \tag{16}$$

$$(x_1 \wedge p_1) \vee x_2 = (x_1 \vee x_2) \wedge (x_2 \vee p_1); \tag{17}$$

$$(x_1 \vee P_1) \wedge (x_2 \vee P_2) = ((x_1 \wedge x_2) \vee (P_1 \wedge P_2)) \vee (x_1 \wedge P_2) \vee (x_2 \wedge P_1); \tag{18}$$

$$(x_1 \wedge P_1) \vee (x_2 \wedge P_2) = ((x_1 \vee x_2) \wedge (P_1 \vee P_2)) \wedge (x_1 \vee P_2) \wedge (x_2 \vee P_1). \tag{19}$$

If we introduce the notations:

$$g_1 = (x_1 \wedge P_1);\ g_2 = (x_2 \vee P_2);\ g_3 = (x_1 \wedge P_1);\ g_4 = (x_2 \wedge P_2);\ x_3 = (x_1 \vee x_2);$$

$$P_3 = (P_1 \vee P_2);\ x_4 = (x_1 \wedge x_2);\ P_4 = (P_1 \wedge P_2);\ g_5 = (x_2 \wedge P_1);\ g_6 = (x_2 \vee P_1);$$

$$g_7 = (x_1 \wedge P_2);\ g_8 = (x_1 \vee P_2);\ g_9 = x_4 \vee P_4;\ g_{10} = x_3 \wedge P_3.$$

Then the relations above may be represented through the following CLV:

$$g_1 \vee g_2 = x_3 \vee P_3 = g_{11};\ g_3 \wedge g_4 = x_4 \wedge P_4 = g_{12}; \tag{20}$$

$$g_1 \wedge x_2 = x_4 \vee g_5 = g_{13};\ g_3 \vee x_2 = x_3 \wedge g_6 = g_{14}; \tag{21}$$

$$g_1 \wedge g_2 = (x_4 \wedge P_4) \vee g_5 \vee g_7 = g_{15};\ g_1 \vee g_2 = (x_3 \vee P_3) \wedge g_6 \wedge g_8 = g_{16}\,. \tag{22}$$

It is proper to define the negation at the complex logical variables similarly to the de Morgan laws, i.e.

$$\neg g_1 = \neg(x_1 \vee p_1) = \neg x_1 \wedge \neg x_2; \tag{23}$$

$$\neg g_2 = \neg(x_2 \wedge P_2) = \neg x_1 \vee \neg x_2. \tag{24}$$

It will be shown that the operations $\vee$, $\wedge$ and $\neg$, defined through relations (14) to (24) for the complex logical variables correspond, both in their real and imaginary parts in particular, to all axioms of Table 1 of the Boolean algebra.

The complement element is defined for the Boolean algebras B_1 and B_2 introduced above in the following way for the real and the imaginary variables:

$$x \vee \neg x = 1;\ x \wedge \neg x = 0;\ P \vee \neg P = 1;\ P \wedge \neg P = 0\,. \tag{25}$$

For the real variables the unit has the truth value (T) and the zero – false (F). For the imaginary variables these values are *i* and $\neg i$ respectively.

By analogy with (25) the complement for the complex variables $g \in G$ will be defined in the following way:

$$g \vee \neg g, \; g \wedge \neg g, \; g \in G, \tag{26}$$

where the first relation in (26) plays the role of unit and the second – of zero.

It may be shown that the axioms from Table 1 are observed for the complex logical variables, and namely:

1. *Associativity* of the disjunction of Axiom 1 directly follows from relation (14).
2. *Commutativity* of disjunction is a corollary of

$$\left(x_1 \vee P_1 \right) \vee \left(x_2 \vee P_2 \right) = \left(x_1 \vee x_2 \right) \vee \left(P_1 \vee P_2 \right) = \left(x_2 \vee P_2 \right) \vee \left(x_1 \vee P_1 \right). \tag{27}$$

3. *Idempotence* of the disjunction for CLV ensues from

$$\left(x \vee P \right) \vee \left(x \vee P \right) = \left(x \vee x \right) \vee \left(P \vee P \right) = x \vee P. \tag{28}$$

4. *Distributivity of disjunction concerning conjunction* follows from (16) and (18).

Axioms 6 and 7 are in conformity with the definitions from (25) and (26). The validity of Axioms 8 to 14 from Table 1 about the conjunction of complex logical variables may be proved in an analogous way.

Like as in propositional logic, it may be shown that the following properties of the Boolean algebra hold for the complex logical variables too:

a) For each g, its complement $\neg g$ is unique;

b) A ”symmetry” of the complement exist, i.e.,

$$\neg\neg g = g. \tag{29}$$

Really, if $g = x \vee p$, then $\neg\neg g = \neg(\neg(x \vee p)) = \neg(\neg x \wedge \neg p) = x \vee p = g$.

c) De Morgan laws are valid also for the complex variables of the Boolean algebra and namely, if $g_1 = x_1 \vee p_1$ and $g_2 = x_2 \vee p_2$, then

$$\neg\left(g_1 \vee g_2 \right) = \neg g_1 \wedge \neg g_2; \tag{30}$$

$$\neg\left(g_1 \wedge g_2 \right) = \neg g_1 \vee \neg g_2. \tag{31}$$

Having in mind that the complex variables and the operations with them submit to the Boolean algebra axioms some new results may be received from the relations 4, and namely:

Proporsition 1: A relation exists

$$\text{T} \wedge i = \text{T}. \tag{32}$$

If in (T Λ i) instead of T its equivalent value from the left hand side of (4) is put, then we receive:

$$\text{T} \wedge i = \left(\text{F} \wedge i\right) \wedge i = \text{F} \wedge \left(i \wedge i\right) = \text{F} \wedge i = \text{T},$$

which confirms (32).

Corollary: The application of negation and of de Morgan laws to both sides of (4) and (32) results in

$$\neg(\text{F}\wedge i) = \neg\text{T}; \; \text{T}\vee\neg i = \text{F}; \tag{33}$$

$$\neg(\text{T}\otimes i) = \neg\text{T}; \; \text{F}\vee\neg i = \text{F}. \tag{34}$$

Relations (4) and (32) may be written in the following way: for each $x \in \{\text{T},\text{F}\}$

$$x \wedge i = \text{T}. \tag{35}$$

Relations (33) and (34) lead to the result:

$$x \neg I = \text{F}. \tag{36}$$

The truth Table 2 for the disjunction from (36) and Table 3 for the conjunction from (35) are of the following kind:

Table 2

x	P	g
T	$\neg i$	F
F	$\neg i$	F

Table 3

x	P	g
T	i	T
F	i	T

It follows from (35) and (36) and the two tables above, that no matter in what state the logical variable x is – T or F its disjunction with $\neg i$ leads always to false, and its conjunction with i – always to true.

Proporsition 2: The following relation exists:

$$\mathrm{T} \vee i = \mathrm{F} \vee i. \tag{37}$$

If in $\mathrm{T} \vee i$ state i is substituted by its equivalence $i = i \vee \neg i$ from (25) keeping in mind (33), then
$\mathrm{T} \vee i = \mathrm{T} \vee (i \vee \neg i) = (\mathrm{T} \vee \neg i) \vee i = \mathrm{F} \vee i$,
which confirms (37).

Corollary: By applying the de Morgan laws separately to both sides of (37) we will receive

$$\neg(\mathrm{T} \vee i) = \mathrm{F} \wedge \neg i;\ \neg(\mathrm{F} \vee i) = \mathrm{T} \wedge \neg i. \tag{38}$$

It follows from (37) and (38) that for each $x \in \{\mathrm{T}, \mathrm{F}\}$

$$x \vee i = \mathrm{T} \vee i = \mathrm{F} \vee i; \tag{39}$$

$$x \wedge \neg i = \mathrm{T} \wedge \neg i = \mathrm{F} \wedge \neg i. \tag{40}$$

The results received provide a possibility for discussion on the values 0 and 1 from (26) and from the axioms 6 and 7 of Table 1 for the complex logical variables.

If the CLV g accepts value $\mathrm{F} \wedge \neg i$ ($\mathrm{T} \wedge i$ respectively) or $g = \mathrm{T} \vee \neg i$ ($\mathrm{F} \vee \neg i$ respectively), then according to Table 2 and Table 3 this complex variable passes into a real one – x, i.e. it should be considered further only in the frame of the classic propositional logic in which the unit (1) corresponds to T and the zero – to F.

Depending on the logical equations of type (1) and (32) to (34) new imaginary variables may arise or through the same equations to turn again into real logical variables. These are processes analogical to those for the complex numbers.

The complex logical values of the type $g = \mathrm{T} \vee i$ ($\mathrm{F} \vee i$ respectively) or $g = \mathrm{F} \wedge \neg i$ ($\mathrm{T} \wedge \neg i$ respectively), which are pretenders for the role of 1 and 0 from (26). In the accepted way of defining the complex logical variables, '$\mathrm{T} \vee i$', may be identified as 1, and '$\mathrm{F} \wedge \neg i$' – as zero. And really, if

$$g = \mathrm{T} \vee i;\ \neg g = \mathrm{F} \wedge \neg i; \tag{41}$$

then from (26) and (41) after respective transformations

$$g \vee \neg g = (\mathrm{T} \vee i) \vee (\mathrm{F} \wedge \neg i) = \mathrm{T} \vee (i \vee (\mathrm{F} \wedge \neg i)) = \mathrm{T} \vee i; \tag{42}$$

$$g \wedge \neg g = (\mathrm{T} \vee i) \wedge (\mathrm{F} \wedge \neg i) = \mathrm{F} \wedge (\neg i \wedge (\mathrm{T} \vee i)) = \mathrm{F} \wedge \neg i. \tag{43}$$

Another conclusion may be also drawn that for the complex variables ($\mathrm{T} \vee i$) corresponds to T for the real variables and ($\mathrm{F} \wedge \neg i$) – to F in the classic propositional logic. This in its turn provides a possibility to construct truth tables for the complex variables from (39) and (40) – Table 4 for the disjunction and Table 5 for the conjunction.

Table 4

№	g_1	g_2	$g_3 = g_1 \vee g_2$
1	$\mathrm{T} \vee i$	$\mathrm{T} \vee i$	$\mathrm{T} \vee i$
2	$\mathrm{T} \vee i$	$\mathrm{F} \wedge \neg i$	$\mathrm{T} \vee i$
3	$\mathrm{F} \wedge \neg i$	$\mathrm{T} \vee i$	$\mathrm{T} \vee i$
4	$\mathrm{F} \wedge \neg i$	$\mathrm{F} \wedge \neg i$	$\mathrm{F} \wedge \neg i$

Table 5

№	g_1	g_2	$g_3 = g_1 \wedge g_2$
1	$\mathrm{T} \vee i$	$\mathrm{T} \vee i$	$\mathrm{T} \vee i$
2	$\mathrm{T} \vee i$	$\mathrm{F} \wedge \neg i$	$\mathrm{F} \wedge \neg i$
3	$\mathrm{F} \wedge \neg i$	$\mathrm{T} \vee i$	$\mathrm{F} \wedge \neg i$
4	$\mathrm{F} \wedge \neg i$	$\mathrm{F} \wedge \neg i$	$\mathrm{F} \wedge \neg i$

Analogically to Table 3 the truth of lines 1 and 4 follow from the idempotence of conjunction and of lines 2 and 3 – from the equivalence in (43) and commutativity of its left hand part.

3. Implication and Equvalence

In a similar way through the CLV ($\mathrm{T} \vee i$) and ($\mathrm{F} \wedge i$) a truth table for the implication can be drawn up, and namely:

Table 6

№	g_1	g_2	$g_3 = g_1 \rightarrow g_2$
1	$\mathrm{T} \vee i$	$\mathrm{T} \vee i$	$\mathrm{T} \vee i$
2	$\mathrm{T} \vee i$	$\mathrm{F} \wedge \neg i$	$\mathrm{F} \wedge i$
3	$\mathrm{F} \wedge \neg i$	$\mathrm{T} \vee i$	$\mathrm{T} \vee i$
4	$\mathrm{F} \wedge \neg i$	$\mathrm{F} \wedge \neg i$	$\mathrm{T} \vee \neg i$

Like with the real logical variables it may be shown that for the implication of the complex logical variables it can be put down:

$$g_3 = g_1 \rightarrow g_2 = \neg g_1 \vee g_2 . \quad (44)$$

And really for row #1 of Table 6 it may be written down:

$\neg(T \vee i) \vee (T \vee i) = (F \wedge \neg i) \vee (T \vee i) = (T \vee i)$,

which follows from (44) and row #3 of Table 4.

Row #2 is confirmed by the idempotent relation:

$\neg(T \vee i) \vee (F \vee i) = (F \wedge \neg i) \vee (F \vee \neg i) = F \wedge \neg i.$

The same may be written down and for row #3 of Table 6:

$\neg(F \wedge \neg i) \vee (T \vee i) = (T \vee i) \vee (T \vee i) = T \vee i$;

and the last row of the same table follows from the second row of Table 4 and (44):

$\neg(F \wedge \neg i) \vee (F \wedge \neg i) = (T \vee i) \vee (F \wedge i) = T \vee i$.

Following the method shown up, the following truth table of the equivalence of the complex logical variables may be drawn up

Table 7

№	g_1	g_2	$g_3 = g_1 \sim g_2$
1	$T \vee i$	$T \vee i$	$T \vee i$
2	$T \vee i$	$F \wedge \neg i$	$F \wedge \neg i$
3	$F \wedge \neg i$	$T \vee i$	$F \wedge \neg i$
4	$F \wedge \neg i$	$F \wedge \neg i$	$T \vee i$

The following relation is true for the complex logical variables, like for the real ones:

$$g_3 = g_1 \sim g_2 = (g_1 \wedge g_2) \vee (\neg g_1 \wedge g_2) . \quad (45)$$

It follows for the first row of Table 7 from the idempotence of conjunction, (42), and (45) that:

$g_3 = (T \wedge i) \vee (F \wedge \neg i) = T \vee i$.

The truth of the second row of Table 7 immediately follows from relations (42), (43),and (45)

$g_3 = ((T \vee i) \wedge (F \wedge \neg i)) \vee (\neg(T \vee i) \wedge \neg(F \wedge \neg i)) = ((T \vee i) \wedge (F \wedge \neg i)) = F \wedge \neg i$.

The truth of the third row of Table 7 may be may be proved in the same way. The fourth row of Table 7 as well as the first row, follow from the idempotence of conjunction, (42), (43), and (45).

$g_3 = ((F \wedge \neg i) \vee (T \vee i)) = T \vee i$.

Table 7 for the implication provides a possibility, like in the classical propositional logic, to deduce the following two rules for inference for the complex propositional logic:

1. Modus Ponens

$$\frac{g_3 = T \vee i,\ g_1 = T \vee i}{g_2 = T \vee i}; \quad (46)$$

2. Modus Tollens

$$\frac{g_3 = T \vee i,\ g_2 = F \vee \neg i}{g_1 = F \wedge \neg i}. \quad (47)$$

4. The imaginary logical variables i and ¬i - illustration

It is not known in what exact interrelation and ratio are the states i or $\neg i$, with T and F respectively, except for the cases of Table 2 and Table 3. The following general conclusion may be conditionally drawn from (4): that i is ‘truer’ from state T of the real variable, and from (33) – that $\neg i$ is ‘falser’ from its state F. These assumptions for the complex logical variables may be illustrated in the following way:

A. Disjunction

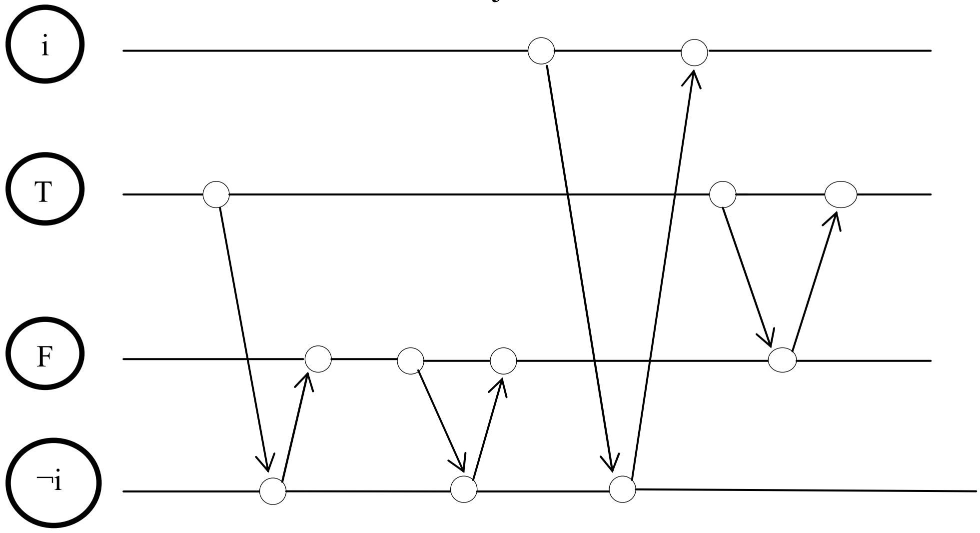

Fig. 1

B. Conjunction

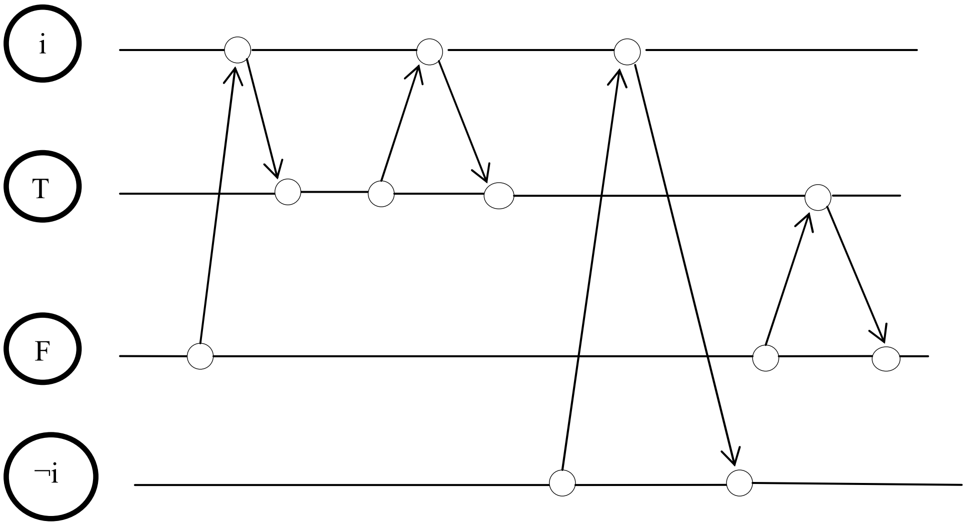

Fig. 2

The initial vertex of each first arrow shows the starting state of the complex variable and its end vertex – its second state which is connected to the first through disjunction/conjunction. The final vertex of the second arrow shows the result of the corresponding logical operation. The last two operations in each of the two figures corresponds to the operations ($i \vee \neg i$) and (T∨F) on Fig. 1 and ($i \wedge \neg i$) and (T∧F) – on Fig.2.

The property of absorbing was shown in (13) to which the real and imaginary logical variables obey. It is also true for the complex logical variables, and namely:

$$g_1 \wedge (g_1 \vee g_2) = g_1; \;\; g_1 \vee (g_1 \wedge g_2) = g_1, \qquad (48)$$

where g_1 and g_2 are CLV.

The check of $g_1 = T \vee i$ and $g_2 = F \wedge \neg i$ for the first of the two equalities (48), considering relation (37) demonstrates:

$g_1 \wedge (g_1 \vee g_2) = (T \vee i) \wedge ((T \vee i) \vee (F \wedge \neg i)) = (T \vee i) \wedge (F \vee i) = T \vee i$.

The second equality of (48) may be checked in a similar way.

Conclusions

The complex propositional logic corresponds to the algebraic structure 'lattice' like the real and imaginary logical variables. It is evident that a series of results may be obtained in the complex propositional logic, which have analogs in the classical propositional logic. Introduction of the complex logical variables, similarly to the complex numbers, provides a possibility to resolve logical equations similar to the one of type (1), which in classic propositional logic has no solution. The abstraction 'complex logical variables' extends to a given degree the abilities of the classic propositional logic. This is of importance for the logic decision making systems, which are actively used in the intelligent systems, as well as in the systems with artificial intelligence. There exist quite good attempts for use of complex numbers in fuzzy logic [6, 7] but there the interpretation is quite different, concerning only the classical complex numbers. In paper the notion 'complex variable' is used as an analogy to describe elements of various algebraic structures in which propositional logic may be described.

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Acad. Vassil Sgurev. Bulgarian Academy of Sciences. Institute of Information and Communication Technologies.
E-mail: vsgurev@gmail.com