Method for Classification of Objects with Fuzzy Values of Features*

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Abstract. The paper is dedicated to the development of classification method for objects, which have their features represented as fuzzy sets. The method is based on computation of the compatibility of the composite premise, which defines an individual class, and the fuzzy features of the objects. The compatibility is represented by means of fuzzy truth values. The results of defuzzification of these values are used to compare the compatibilities and thus to determine the class of the object.

Keywords: extension principle, fuzzy truth value, linguistic variable.

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Introduction

Many of proposed classification methods are based on soft computing techniques, such as artificial neural networks [1] or fuzzy systems [2–4]. These methods assume that the features of the classified objects take numerical values only. This paper proposes a classification method, which is capable of classifying objects, the features of which are represented as terms of linguistic variables [5]. An important advantage of this method is the inference within a single space of truthfulness, which reduces the computational complexity of the inference down to polynomial. This is particularly important in problems such as gene classification based on data mining, which produce fuzzy rules with hundreds of premises [6].

The article consists of three sections. The problem of classification is stated in the first section. The second section defines the fuzzy truth value of the compound premise of rule, which describes a class, with respect to an object, defined by a set of terms of linguistic variables. The third section introduces the principle of determination of the object's class.

1. Statement of the Classification Problem

Let $[x_1, x_2, \ldots, x_n]$ denote the vector of features of an object q'. Each of these features is assigned a linguistic variable, terms of which are the values of the feature. Then the object q' can be defined in following way:

$$\langle x_1 \text{ is } A_1' \text{ and } x_2 \text{ is } A_2' \text{ and } \ldots \text{ and } x_n \text{ is } A_n' \rangle, \quad (1)$$

where $A_i' \subseteq X_i, i = \overline{1,n}$ are the fuzzy sets, which formalize the terms of the linguistic variables.

Let us denote the set of classes as $\Omega = \{\omega_1, \omega_2, \ldots, \omega_n\}$. The classes are defined by means of a fuzzy rule base of the following form:

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$$
\begin{aligned}
R_k:\ &\text{If } x_1 \text{ is } A_{11}^k \text{ and } x_2 \text{ is } A_{12}^k \text{ and} \ldots \text{and } x_n \text{ is } A_{1n}^k,\\
&\text{or } x_1 \text{ is } A_{21}^k \text{ and } x_2 \text{ is } A_{22}^k \text{ and} \ldots \text{and } x_n \text{ is } A_{2n}^k,\\
&\vdots\\
&\text{or } x_1 \text{ is } A_{P_k 1}^k \text{ and } x_2 \text{ is } A_{P_k 2}^k \text{ and} \ldots \text{and } x_n \text{ is } A_{P_k n}^k,\\
&\text{then } y \text{ is } \omega_k,\ k=\overline{1,N},
\end{aligned}
\tag{2}
$$

where N is the number of fuzzy rules, $A_{ji}^k \subseteq X_i$, $j=\overline{1,P_k}$, $i=\overline{1,n}$ are fuzzy sets, which formalize the terms of the linguistic variables used to define the features of the object q'.

The classification problem consists in assigning the object q' represented in the form (1), to one of the predefined classes in Ω, i.e. mapping

$$\langle x_1 \text{ is } A_1' \text{ and } x_2 \text{ is } A_2' \text{ and } \ldots \text{ and } x_n \text{ is } A_n' \rangle \to$$
$$\to y \in \{\omega_1, \omega_2, ..., \omega_N\}.$$

2. Definition of Fuzzy Truth Value (FTV) of the Compound Premise of Rule Rk with Respect to q'

Let us define the FTV of a fuzzy set A with respect to another fuzzy set A', which have membership functions $\mu_A(x)$ and $\mu_{A'}(x)$ correspondingly. Let us consider the truth-functional modification principle [4], which can be written in the form

$$\mu_{A'}(x) = \mu_{CP(A,A')}(\mu_A(x)), \tag{3}$$

where $\mu_{CP(A,A')}(\cdot)$ is the membership function of the FTV. $CP(A,A')$ also represents the compatibility of a fuzzy set A with respect to fuzzy set A', where A' is considered true [6]:

$$\mu_{CP(A,A')}(v) = \sup_{\substack{\mu_A(x)=v\\ x\in X}} \{\mu_{A'}(x)\}.$$

Let us use variable v instead of x in (3), denoting $v=\mu_A(x)$. Then we shall obtain the following:

$$\mu_{A'}(x) = \mu_{CP(A,A')}(\mu_A(x)) = \mu_{CP(A,A')}(v). \tag{4}$$

So the FTV $CP(A,A')$ precisely describes the relative position of the fuzzy set A with respect to the fuzzy set A'. An important advantage of using the FTV is the containment of fuzzy relation between the fact A' and the premise A. A simpler approach is the use of the possibility measure [6]:

$$\Pi_{A'}(A) = \sup_{x\in X} (\mu_A(x)\ \mathrm{T}\ \mu_{A'}(x)),$$

which reduces the information about the relation between the fact and the premise down to a scalar value. If the membership functions $\mu_A(x)$ and $\mu_{A'}(x)$ are Gaussian curves, then the membership function of the FTV can be determined analytically as described in [5]. If $\mu_A(x)$ and $\mu_{A'}(x)$ are piecewise-linear, then FTV can be computed according to an efficient algorithm proposed in [7].

Let us determine the FTV of j-th premise of rule (2) with respect to the object q' definition, represented in as (1). Denote j-th premise of rule R^k as

$$\boldsymbol{A_j^k} = A_{j1}^k \times A_{j2}^k \times \ldots \times A_{jn}^k,$$

wherein

$$\mu_{\boldsymbol{A_j^k}}(\boldsymbol{x}) = \mu_{\boldsymbol{A_j^k}}(x_1, x_2, ..., x_n) = \underset{i=\overline{1,n}}{\mathrm{T_1}}\ \mu_{A_{ji}^k}(x_i), \tag{5}$$

where $\boldsymbol{x}\in\boldsymbol{X} = X_1\times X_2\times\ldots\times X_n$ and $x_i \in X_i$. Let us also represent the definition of the object q' as

$$\boldsymbol{A'} = A_1' \times A_2' \times \ldots \times A_n',$$

wherein

$$\mu_{\boldsymbol{A'}}(\boldsymbol{x}) = \mu_{\boldsymbol{A'}}(x_1, x_2, ..., x_n) = \underset{i=\overline{1,n}}{\mathrm{T_2}}\ \mu_{A_i'}(x_i), \tag{6}$$

where $\mathrm{T_1}$ and $\mathrm{T_2}$ are arbitrary t-norms.

It is required to prove that

$$CP(\boldsymbol{A_j^k}, \boldsymbol{A'}) = \underset{i=\overline{1,n}}{\tilde{\mathrm{T}}_1}\ CP(A_{ji}^k, A_i'), \quad j=\overline{1,P_k}, \tag{7}$$

where $\tilde{\mathrm{T}}_1$ is an extended by means of the extension principle n-ary t-norm [8], which is defined as

$$\mu_{\underset{i=\overline{1,n}}{\tilde{\mathrm{T}}_1} CP(A_{ji}^k, A_i')}(v) = \sup_{\substack{\underset{i=\overline{1,n}}{\mathrm{T_1}} v_i = v\\ (v_1,\ldots,v_n)\in[0;1]^n}} \{\underset{i=\overline{1,n}}{\tilde{\mathrm{T}}_2}\ \mu_{CP(A_{ji}^k, A_i')}(v_i)\}. \tag{8}$$

According to the definition of fuzzy truth value [5]

$$\mu_{CP(\boldsymbol{A_j^k}, \boldsymbol{A'})}(v) = \sup_{\substack{\mu_{\boldsymbol{A_j^k}}(x_1, x_2, ..., x_n)=v\\ (x_1, x_2, ..., x_n)\in\boldsymbol{X}}} \{\mu_{\boldsymbol{A'}}(x_1, x_2, ..., x_n)\}, \tag{9}$$

which has an exponential computational complexity $\mathrm{O}(|X_1\times X_2\times\ldots\times X_n|)$, according to (5) and (6) can be written as:

$$\mu_{CP(\boldsymbol{A_j^k}, \boldsymbol{A'})}(v) = \sup_{\substack{\underset{i=\overline{1,n}}{\mathrm{T_1}} \mu_{A_{ji}^k}(x_i)=v\\ (x_1, x_2, ..., x_n)\in\boldsymbol{X}}} \{\underset{i=\overline{1,n}}{\mathrm{T_2}}\ \mu_{A_i'}(x_i)\}.$$

Let us use variable v_i instead of x_i according to (4), i.e.

$$v_i = \mu_{A_{ji}^k}(x_i) \quad \text{and} \quad \mu_{A_i'}(x_i) = \mu_{CP(A_{ji}^k, A_i')_i}(v_i).$$

Thus we shall obtain (8) and (7) correspondingly.

Expression (7) at the level of membership functions can be written as

$$\mu_{CP(\boldsymbol{A_j^k},\boldsymbol{A'})}(v) = \underset{i=\overline{1,n}}{\tilde{\mathrm{T}}_1} \mu_{CP(A_{ji}^k,A_i')}(v_i) =$$

$$= \left(\left(\mu_{CP(A_{j1}^k,A_1')}(v_1) \,\tilde{\mathrm{T}}_1\, \mu_{CP(A_{j2}^k,A_2')}(v_2) \right) \tilde{\mathrm{T}}_1 \right. \quad (10)$$

$$\left. \tilde{\mathrm{T}}_1\, \mu_{CP(A_{j3}^k,A_3')}(v_3) \right) \tilde{\mathrm{T}}_1 \ldots \tilde{\mathrm{T}}_1\, \mu_{CP(A_{jn}^k,A_n')}(v_n).$$

For example, binary extended t-norm is defined as:

$$\mu_{CP(\boldsymbol{A_j^k},\boldsymbol{A'})}(v) = \underset{i=\overline{1,2}}{\tilde{\mathrm{T}}_1} \mu_{CP(A_{ji}^k,A_i')}(v_i) =$$

$$= \sup_{\substack{v_1 \mathrm{T}_1 v_2 = v \\ (v_1, v_2) \in [0;1]^2}} \{ \mu_{CP(A_{j1}^k,A_1')}(v_1) \,\mathrm{T}_2\, \mu_{CP(A_{j2}^k,A_2')}(v_2) \}.$$

The latter expression has the order of computational complexity equal $O(|v|^2)$. Thus, the computational complexity of the expression (10) has the order of $O(n|v|^2)$. Therefore, when the conditions (5) and (6) are met, exponential complexity of computing $\mu_{CP(\boldsymbol{A_j^k},\boldsymbol{A'})}(v)$ (9) is reduced down to polynomial.

When $CP(\boldsymbol{A_j^k}, \boldsymbol{A'})$ is computed for each premise $j = \overline{1, P_k}$, rule R^k can be represented as

$$R_k : \text{If } CP(\boldsymbol{A_1^k}, \boldsymbol{A'}) \text{ or } CP(\boldsymbol{A_2^k}, \boldsymbol{A'}) \text{ or} \ldots \text{or } CP(\boldsymbol{A_{P_k}^k}, \boldsymbol{A'}),$$

$$\text{then } y \text{ is } \omega_k,\ k = \overline{1, N}. \quad (11)$$

The antecedent of the rule (11) represents an expression, which consists of fuzzy truth values $CP(\boldsymbol{A_j^k}, \boldsymbol{A'})$, $j = \overline{1, P_k}$, defined on different unit intervals and united together via linguistic joint "or". This joint is formalized as a t-conorm.

In order to compute FTV of the compound premise of the rule R^k extension principle is also used, according to which:

$$CP(\boldsymbol{A^k}, \boldsymbol{A'}) = \underset{j=\overline{1,P_k}}{\tilde{\mathrm{S}}} CP(\boldsymbol{A_j^k}, \boldsymbol{A'}), \quad k = \overline{1, N},$$

where $\tilde{\mathrm{S}}$ is an extended by the extension principle P^k-ary t-conorm, which is defined as

$$\mu_{CP(\boldsymbol{A^k},\boldsymbol{A'})}(v) = \sup_{\substack{\underset{j=\overline{1,P_k}}{\mathrm{S}} v_j = v \\ (v_1,\ldots,v_{P_k}) \in [0;1]^{P_k}}} \{ \underset{j=\overline{1,P_k}}{\mathrm{T}} \mu_{CP(\boldsymbol{A_j^k},\boldsymbol{A'})}(v_j) \}. \quad (12)$$

Computational complexity of the expression (12) has order of $O(P^k|v|^2)$, as well as (10) does.

3. Determination of Class of Object q'

$\mu_{CP(\boldsymbol{A^k},\boldsymbol{A'})}(v)$ is the membership function of FTV, which represents the compatibility of the premise of rule (2), which defines a class ω_k, with respect to the object q', the features of which are represented as a set of terms of linguistic variables. This compatibility is formally represented as a fuzzy set, defined on a unit interval. In order to determine the class of the object q' it is proposed to use method the described in [9]. According to this method, every fuzzy set defined on [0;1] is associated a value

$$F(D) = \frac{1}{\alpha_{\max}} \int_0^{\alpha_{\max}} M(D_\alpha) d\alpha, \quad (13)$$

where $\alpha_{\max}$ is the maximum membership degree of a fuzzy set D, D_α is α-level set, i.e. a set of the form

$$D_\alpha = \{ v \mid v \in [0;1] \wedge \mu_D(v) \geqslant \alpha \},$$

$M(D_\alpha)$ – is the cardinality of α-level set, which, in case of discrete representation of $\mu_D(v)$ is defined as

$$M(D_\alpha) = \frac{1}{\ell} \sum_{k=1}^{\ell} v_k, \quad v_k \in D_\alpha.$$

Let us consider the FTV as a linguistic variable:

$$\langle FTV, T, [0;1], G, M \rangle,$$

where T is a term-set, [0;1] is the domain of definition, G is a syntactic rule, M is a semantic rule.

Let the term-set take the following values:

$$T = \{\text{«true»},\ \text{«false»},\ \text{«absolutely true»},$$

$$\text{«absolutely false»},\ \text{«unknown»}\}.$$

The semantics of the terms is defined as:

$$M[\text{«true»}] = \int_0^1 v/v;$$

$$M[\text{«false»}] = \int_0^1 1-v/v;$$

$$M[\text{«absolutely true»}] = 1/1 + \int_0^1 0/v;$$

$$M[\text{«absolutely false»}] = 1/0 + \int_0^1 0/v;$$

$$M[\text{«unknown»}] = \int_0^1 1/v.$$

If the expression (13) is used to obtain a numerical assessment of these values of FTV, then, as shown in [9], they are following:

$$F(\text{«true»}) = \tfrac{3}{4};$$

$$F(\text{«false»}) = \tfrac{1}{4};$$

$$F(\text{«absolutely true»}) = 1;$$

$$F(\text{«absolutely false»}) = 0.$$

$$F(\text{«unknown»}) = \tfrac{1}{2}.$$

Therefore numerical values correlate to the verbal values of terms of linguistic variable FTV. Let us use this method to determine the class of the object q'. A class with the greatest value of the assessment is considered as the class of q', i.e.

$$y = \omega_s, \quad \text{where} \quad s = \arg\max_{k=\overline{1,N}} \{F_{\omega_k}(CP(\boldsymbol{A^k}, \boldsymbol{A'}))\},$$

where $F_{\omega_k}(CP(\boldsymbol{A^k}, \boldsymbol{A'}))$ denotes the numerical assessment of the compatibility of the description of class ω_k by the rule R^k with respect to the definition of the object q'.

The network structure of the described method, consisting of multiple layers, is depicted in Fig. 1. The first layer computes the compatibility of each fuzzy input of the corresponding premise to the fuzzy set, describing the feature of the object q'. Second and third layers implement the convolution of FTVs for each premise in rule base (2), the result of which defines the compatibility of j-th premise with respect to the object q', represented in the form (1). Fourth and fifth layers computethe compatibility of the entire premise of rule (2) with respect to object q' by means of the extended by extension principle P^k-ary t-conorm. The last layer denotes the computation of numerical assessment of the fuzzy set $CP(\boldsymbol{A^k}, \boldsymbol{A'})$ using the method of comparing of fuzzy sets defined on unit interval.

Conclusion

The article proposes a classification method for case if, in contrast to the traditional approach, non-singleton (NS) fuzzification [10] is applied to the features of the objects. NS fuzzification is used in fuzzy systems if the measurements of object features are inaccurate or uncertain (due to measurement errors, degradation of sensors etc.), or when the features are estimated by the terms of linguistic variables. NS fuzzification models measurements or terms such as fuzzy numbers or more general fuzzy sets, thus regardless of the nature of uncertainty they are handled within the fuzzy sets theory.

The use of fuzzy truth value, which preserves fuzziness and accurately describes the relative position of one fuzzy set with respect to another, has been proposed to estimate the compatibility of terms with

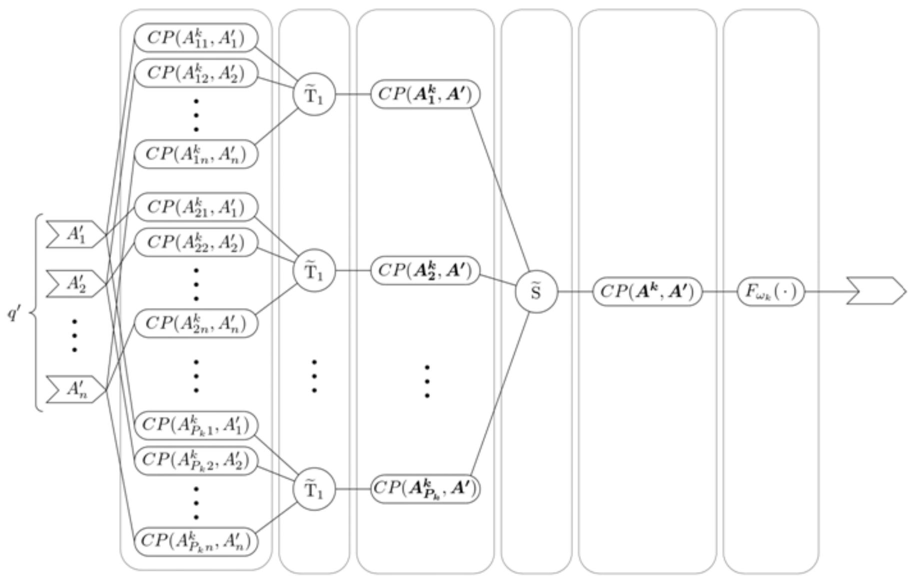

Fig. 1. Network structure of the classification method

respect to input values. In contrast to the application of the possibility measure, it doesn't reduce the information about the relation between the fact and the premise to a scalar value. A proof about the reduction of the computational complexity of determining the FTV between multidimensional membership functions down to polynomial is given in the article. This makes it possible to solve such classification problems for objects with a large number of attributes. A known method of comparison of fuzzy sets defined on a unit interval is used to determine the class of the object. Development of learning algorithms using parallel technologies in accordance with the network structure is the subject of further research.

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