Mathematical Modeling for Intelligent Transport Infrastructure Development

T. M. Gataullin[I], E. S. Pleshakova[III], P. G. Bylevskiy[II], A. V. Osipov[III], S. T. Gataullin[III,IV]

[I] New Economic Association, Moscow, Russia
[II] Moscow State Linguistic University, Moscow, Russia
[III] MIREA Russian Technological University, Moscow, Russia
[IV] Central Economics and Mathematics Institute of the Russian Academy of Sciences, Moscow, Russia

Abstract. In order to increase the reliability of feasibility studies for the development of intelligent transport infrastructure in the context of the sustainable development of megacities, it is necessary to more carefully take into account the actual operating conditions of single-level intersections of traffic flows.The solution of this problem requires the use of advanced economic and mathematical tools. The mathematical description is a modification of the fundamental results of E. Borel and F. Haight related to the simplest flow of events. Applications of this method will allow both assessing the economic feasibility of expanding the existing transport infrastructure and designing new transport facilities. This approach can be applied at the stage of the initial quantitative assessment of the feasibility of replacing a one-level intersection of traffic flows with a multi-level interchange to avoid economic losses.

Keywords: mathematical modeling, sustainable development, digital economy, intelligent transport infrastructure, queuing theory.

DOI 10.14357/20718632240308 **EDN** OYISFY

Introduction

Mathematical and economic-mathematical tools are in most cases inherent in the creation of new methods using the latest equipment, improving the automation of cross-traffic, making the construction of multi-level interchanges redundant, requiring many resources, many months and years and high financial costs. Automation of traffic control at single-level intersections of the road network in a significant number of cases successfully competes with the construction of multi-level interchanges. In intelligent automation, one of the leading roles belongs to mathematical methods, and in improving its efficiency - to economic and mathematical tools.

Intelligent traffic light control systems make transportation faster and more efficient and predictable, reduce travel time and downtime, reduce harmful gaseous emissions, noise and traffic incidents much easier and cheaper. However, the endless regional diversity of road networks and intersections, types, models and number of vehicles makes it difficult to implement standard solutions, requiring research and development of new methods for optimizing automated traffic control at one-level intersections.

The latest research, method developments and proposed solutions in recent years can improve automated traffic control at single-level intersections directly or indirectly to varying degrees. Direct methods concern the use of new equipment, devices, and software, Internet of Things (IoT) technologies, big data generation and analysis, and artificial intelligence [1]. Indirect methods include advanced evaluation and calculation techniques, grading levels, control, statistics and analysis, efficiency calculations [2], including safety aspects and social benefits [3]. Also, indirect methods include improvements in the architecture of transport traffic management [4], including the reduction of travel time uncertainty based on “Ex-post” indicators [5], as well as new methods of computer modeling and forecasting, integration of air transport capabilities, including "unmanned".

The perspective structure of traffic distribution in UAM urban airspace is based on flow modeling and structuring of a three-dimensional (3D) two-way network. The motion optimization problem is solved by means of a linear dynamic system (LDS), a two-phase approach combining simulated annealing (SA) and the Dafermos algorithm (DA). The new complexity metric is defined as an objective function that takes into account the dynamic structure of the flow, workload and operational efficiency [6].

The expansion of the use of aircraft in urban transport not only requires new regulatory solutions, but also creates opportunities for improving the automated control of ground traffic. The dynamic collection of growing volumes of increasingly diverse data generated requires increasingly faster processing and analysis, approaching real time. For video monitoring of traffic streams, a high-performance solution using machine vision and a neural network (YOLOv4) is proposed. The number and dimensions of vehicles stopped before the stop line, the dynamics of braking before and accelerating after the intersection are recorded [7-8].

The spread of the Internet of things (IoT) is accompanied by the expansion of communications and the integration of various objects of vehicles and road infrastructure. The rapid growth of a variety of sensors with different characteristics makes it difficult to classify and integrate the data they generate from an ever larger list of species. The developers make a critical review of existing traditional classifications and propose an improved taxonomy of road traffic network traffic [9].

Intelligent transport systems such as VANET (Vehicular Adhoc Networks) serve as an integrating solution for data exchange between vehicle speed adapters and road infrastructure. Models of mobility, simulation and prediction of traffic distribution at intersections are developed on the principles of stochastic processes and queuing theory. The queuing system is analyzed as a continuous-time Markov chain (CTMC), steady state probabilities calculations produce performance measures for various scenarios [10].

Accounting for vehicle power and fuel type, the microemission model of harmful emissions, and the K-means clustering analysis method make it possible to integrate stationary (according to city sensors) and dynamic (according to bus driving profiles) load indicators, to determine traffic scenarios [11-13].

An advanced road traffic management methodology has been developed for an unknown, time-varying fundamental diagram (FD) with variable traffic composition, connected and automated vehicles (CAVS) with different driving characteristics. The reference model adaptive control maximizes problem area throughput by being integrated into a control scheme including a linear quadratic cumulative regulator to control traffic with a percentage of CAVs. The effectiveness of the proposed approach is illustrated by simulation experiments with a multi-lane macroscopic traffic flow model that takes into account the reduction in throughput [14]

The multiple regression equations included all considered types of vehicles, allowing varying predictive estimates for different traffic patterns. The categorical traffic flow model at the intersection, created on the basis of fuzzy logic methods, made it possible to present 3D capacity estimates in the graph, taking into account some uncertain factors of the traffic flow [15-16]

The modernization of cross traffic control centers is carried out using specialized digital twins and artificial intelligence, based on the analysis of existing solutions, a reference model has been developed [17]. Also, adapted Building Information Modeling (BIM) tools are useful as a methodological basis for traffic analysis and road intersection design modeling [18].

New methods make it possible to successfully implement automation in non-standard cases (for example, when reducing the number of lanes on bridges and tunnels) [19]. Dedicated automation solutions offer the opportunity to improve traffic management in developing countries and regions where the creation of a modern road network is a distant future [20], as well as at minor intersections where large-scale construction is not economically feasible.

The potential of automation can be applied even at non-signalled intersections, regulated based on the perceptions of vehicle drivers. Parameters that cannot be directly measured in the field can be estimated using a simulation model with the combined use of a functionally connected artificial neural network (FLANN) and differential evolution (DE) [21-23]. Systematic elimination of dispersed local "bottlenecks" by new intelligent methods increases the overall efficiency of the road transport network at the regional, national and international levels.

1. Related works

In research on transport issues, scientists often use probability theory, queuing theory and other branches of applied mathematics, as well as various simulation methods for skipping flows in transport networks. Despite a great contribution to the development of the probabilistic approach was made, such formulas have not received wide practical application. The fundamental results associated with the simplest flow of events, obtained by E. Borel [24], were used by Saaty [25-27] and Haight [26] to analyze the operation of traffic control in which the normal position of the traffic light is closed. Haight used the same design to analyze the work in other modes. The modification of the method proposed by the authors is that additional restrictions are imposed on the operation of traffic lights, making the entire construction a Markov random process. Under these conditions, one can quantify the stationary probability and calculate the desired characteristics. This method will allow us to estimate the economic losses corresponding to transport delays. By embedding the proposed method into the algorithms of the intelligent traffic flow control system, it is possible to solve a whole class of problems related to an accurate assessment of the economic feasibility of modifying the transport infrastructure, for example, replacing single-level intersections with multi-level ones. You can also apply the method when designing a transport network and determining the number of intersections of traffic flows following from one part of the city to another. At the same time, the proposed method can be an alternative and a basis for comparison with more time-consuming and expensive methods of modeling presented in the introduction.

2. Materials and Methods

It should be noted that answers can be obtained under fairly strict assumptions about the mode of operation of the traffic light, the conditions for the passage of cars through it, and the nature of their flow. The main concept in the following is the simplest flow of events - in this case, the flow of cars arriving at a traffic light. The famous mathematician E. Borel obtained fundamental results in this direction as early as the middle of the last century. This design was used to analyze the operation of a traffic light, in which the normal position of the traffic light is closed. Frank Haightfinds conditional probabilities $f(z,x)$ that at the moment the red light comes on, waiting for the green light $Z = z$ cars waiting for the green light, provided that at the moment the previous green light came on there were X = x cars, i.e. $f(z,x) = P(Z = z/X = x)$. [25]

The probability that in the simplest flow of intensity λ events per unit time in period t, k events will occur is equal to

$$P(t,k) = \frac{(\lambda t)^k}{k!} e^{-\lambda t} \quad (1)$$

We also note that the simplest flow is also characterized by the fact that the time interval between two successive events is distributed according to an exponential law with the same parameter λ , the average time between events is $1/\lambda$.

2.1. The normal position of the traffic light is closed

Suppose in the simplest flow of cars (cars) - intensity λ, the passage of a traffic light is allowed at regular intervals T (i.e., the time the traffic light is occupied by one car all the time). The normal position of the traffic light is closed, it opens for a time not exceeding α and closes if there are no waiting cars in front of the traffic light. The normal position of the traffic light is closed, it opens for a time not exceeding α and closes if there are no waiting cars in front of the traffic light. During the specified time open through it passes no more than some constant number of cars N, consequently, $T = \alpha/N$. Denote $\rho = \lambda T$ so there ρ is the average number of arriving cars in the stream during the time one car passes through the traffic light. Note that the approximate relation $\rho \approx \alpha\lambda/N$.

It is convenient to take a new unit of time - namely, the time for which the car passes the traffic light, i.e. T . This is equivalent to counting $\lambda = 1$. Then ρ gets the meaning of the intensity of the flow of cars arriving at the traffic light in a (new) unit of time, and at the same time ρ approximately equal to α/N is the number of units of time it takes the car to pass the traffic light. (In fact, the transition to new notation is the transition to dimensionless quantities). These notations are taken from the work of F. Haight, who, in turn, used the necessary results of E. Borel.

Let us consider the Borel construction, which will be needed in what follows. Let $\{A_i\}$ events of the simplest flow on the real axis, the intensity of the flow is equal to ρ (again, we recall that λ it is considered equal to 1). Let's construct some sequence of intervals. Let A_1 the first event of the thread after the start of counting, i.e. 0. Set aside from A_1 length segment ρ and its end is denoted by a dot B_1 If in the segment A_1B_1 there are no flow points, then the desired sequence is constructed, and it consists of one interval A_1B_1 length ρ. If the specified interval contains α_1 points - events of the flow, then we postpone from the point B_1 length interval $\alpha_1\rho$ get a point B_2. If in this interval B_1B_2 there are no flow points, then the desired sequence is constructed and it consists of $1 + \alpha_1$ length intervals ρ every. And so on, until in some interval $B_{k-1}B_k$ is α_k flow points, and in the interval B_k, B_{k+1} total length $\alpha_k\rho$ there are no flow points, then the desired sequence is constructed and it consists of $1 + \alpha_1 + \ldots + \alpha_k$ length intervals ρ each (in the interval B_{k-1}, B_k is α_k points). The sequence can also be of infinite length. Then you can construct the next sequence in the same way, taking the first point of the flow to the right of the points of the constructed sequence as the first point of the new sequence.

What is the probability p_n that an arbitrary sequence thus constructed consists of $n = 1 + \alpha_1 + \ldots + \alpha_k$length intervals ρ every. Through rather complicated reasoning, using the theory of power series and information from the theory of functions of a complex variable, Borel finds that

$$p_n = \frac{n^{n-2}}{(n-1)!}\rho^{n-1}e^{-n\rho} \qquad (2)$$

For some variants of parameter relations, Borel also finds the probability that the desired sequence has an infinite length, but the most interesting case is when this probability is equal to 0, which we restrict ourselves to. Further, Borel, somewhat generalizing his construction, lays aside the point B_1 not at a distance of one length interval ρ from the point A_1 and at a distance of total length r such intervals, each length ρ . In this case, the probability p_n turns out to be equal

$$p_n = r\frac{n^{n-r-1}}{(n-r)!}\rho^{n-r}e^{-n\rho} \text{ for } n = r, r+1, \ldots. \qquad (3)$$

This Borel construction can be connected with a one-level intersection of traffic flows as the following. Suppose that at the moment the green signal turned on, there was a queue in front of the traffic light r vehicle. During the passage through the traffic lights of these r vehicle - this time is $r\rho$ some more will come α_1 vehicle; when these traffic lights pass α_1 cars - in time $\alpha_1\rho$ some more will arrive at the traffic light α_2 cars etc. up to some k -th step for which- length $\alpha_k\rho$ not a singlecar will arrive at the traffic light, the queue will be reset, as a result of which it will immediately turn on the red light (that's what it means that its normal position is closed!). Then the probability that these intervals $r\rho + \alpha_1\rho + \ldots + \alpha_k\rho$ combined together make a sequence of length $n\rho$ those. together they make n spans long ρ each is equal p_n see formula (3). After such a period of time, not a single car will remain in front of the traffic light and it closes, thus, by the next red signal, there will be no cars waiting in front of the traffic light.

The probability of formula (3) F. Haight means $R(n,r)$ Wherein n, r must still satisfy some natural conditions (for example, r should be no more N otherwise, the queue cannot be reset for the entire duration of the green signal).

Let $f(z,x)$ there is a conditional probability that at the moment the red light comes on, there is a $Z = z$ cars, provided that at the moment the previous green light came on, there was $X = x$ vehicles, i.e. $f(z,x) = P(Z = z / X = x)$ Using Borel's results above, Haight shows that

$$f(z,x) = \begin{cases} \frac{(\alpha\lambda)^{z-(x-N))}e^{-\alpha\lambda}}{(z-(x-N))!}, x > N \\ \sum_{j=x}^{N} R(j;x), z = 0, x \le N \\ e^{\rho z}[R(N+z;x) - \sum_{j=1}^{z-1} R(z;j)f(j;x)], z > 0, x \le N \end{cases} \quad (4)$$

In principle, taking into account the changed notation, these formulas can be rewritten as follows (just assume that $\lambda = 1$ Then $\alpha\lambda$ approximately equal to $N\rho$).

$$f(z,x) = \begin{cases} \frac{(N\rho)^{z-(x-N))}e^{-N\rho}}{(z-(x-N))!}, x > N \\ \sum_{j=x}^{N} R(j;x), z = 0, x \le N \\ e^{\rho z}[R(N+z;x) - \sum_{j=1}^{z-1} R(z;j)f(j;x)], z > 0, x \le N \end{cases} \quad (5)$$

The first equation shows that during the time the green signal is active, N vehicle but all vehicle are included $x > N$ will not have time to pass, will remain in front of him $x - N$ means in time $\alpha = N\rho$ should arrive before moving $z - (x - N))$ vehicle, the probability of this for the simplest flow is calculated by formula (1).

Now consider the equation of formula (6). In him

$$R(n;r) = r\frac{n^{n-r-1}}{(n-r)!}e^{-n\rho}\rho^{n-r} \quad (6)$$
$$(n = r, r+1, \ldots).$$

So that by the time the red light turns on there are no cars left in front of the traffic light, it is necessary that the queue is reset to zero in some time, no more N spans long ρ. And in less time $x\rho$ it cannot reset to zero - after all, they must pass x queuing cars!

The third equation can be interpreted as follows. First for $z = 1$ we have -$R(N+1;x) = f(1;x)e^{-\rho}$. By the time the green light came on, they were waiting in line. x cars, even if only 1 car remained by the time of the red signal, $R(N+1,x)$ - is the probability that $x + \alpha_1 + \ldots + \alpha_k = N$ (unit of measurement is a span of length ρ); for the next interval ρ not a single car should arrive - because for $N+1$ intervals, the queue should “zero out”: the probability of this is equal to $e^{-\rho}$. Similarly, to this reasoning, we have, in the general case, we can write:

$$R(N+z,x) - f(z,x)e^{-z\rho} = \sum_{j=1}^{z-1} f(j;x) \cdot R(z;j) \quad (7)$$

An analogue of the total probability formula has been obtained, but it must be skillfully used. For example, when $z = 1$ the sum on the right should be considered equal to 0 (Fig. 1).

2.2. The first possible mode

The normal position of the traffic light is open. Suppose that the flow of cars is the simplest intensity λ =1 vehicle per unit of time (for simplicity, we keep the agreements of the previous paragraph). Passage of cars past the traffic light is allowed at regular intervals T. The traffic light is closed according to the conditions of train movement along the haul for a time not exceeding α. Other assumptions can also be refined.

2.3. The second possible mode

The traffic light mode is regular. traffic light for a while t_r lights red (red), then for a while t_g green (green) and the lengths of these gaps are constant. Such a regime is also possible and needs to be studied.

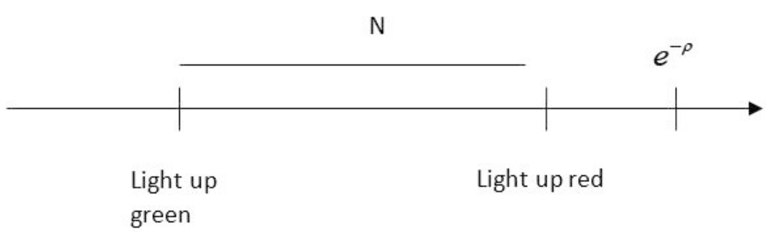

Fig. 1. Scheme for formula (5)

2.4. The third possible mode

The traffic light mode is random. The traffic light is red for some time, then green for some time, the lengths of these intervals are random and subject to some kind of distribution laws, for example, exponential, with their own parameters g, r - respectively: then the average green light burning time is equal to $1/g$, red - $1/r$; it is also obvious that $1/g+1/r=1$.

Recall that if a random (s.) quantity (v.) ξ has an exponential distribution with parameter λ Then $P(\xi > a) = e^{-\lambda a}$, expected value ξ equals $1/\lambda$.

In what follows, the characteristic property of the exponential distribution is used.

A characteristic property of the exponential distribution. Let s. v ξ has an exponential distribution, then $P(\xi>a+b/\xi>b)=P(\xi>a)$. This characteristic property can be formulated in words as follows: if, for example, ξ this is the burning time of the green (or red) signal, then the distribution of the lagging burning time at any moment of burning is also indicative, and the parameter of this distribution remains the same. This property is called characteristic because if the distribution has it, then it is necessarily exponential. The green light mode of a real traffic light at a railway crossing approximately has this property.

Proof. See, for example in [28].

In what follows, we use the following well-known property about exponential random variables, which weformulate as Proposition 1.

Proposition 1. Let ξ_1, ξ_2 independent r.v., exponentially distributed with parameters, respectively, λ_1, λ_2. Then

$$P(\xi_1 < \xi_2) = \frac{\lambda_1}{\lambda_1 + \lambda_2},\ P(\xi_2 < \xi_1) = \frac{\lambda_2}{\lambda_1 + \lambda_2}$$

Proof. See, for example in [28].

Proposition 2. For any natural x $f(0;x) = \sum_{j=x}^{\infty} R(j;x) \cdot e^{-g\rho j} f(0;0) = f(0;0) \sum_{j=x}^{\infty} R(j;x)\, e^{-g\rho \vec{\ } j} = f(0;0) R(x)$

Where $R(r) = \sum_{n=r}^{\infty} R(n;r)\, e^{-gn\rho} = \sum_{n=r}^{\infty} r \frac{n^{n-r-1}}{(n-r)!} e^{-(n\rho + gn\rho)} \rho^{n-r}$

Proof. As you know, see above $R(j;x)$ there is a possibility that j intervals of length each ρ queue of x cars in front of the traffic lights will be reset to zero for the first time. But for this, the green light must still be on at least $\rho\, j$ units of time. The probability of this is $e^{-g\, j\rho}$ After that we fix $x = 0$ and due to the properties of the exponential distribution, we obtain the probability $f(0;0)$

Proposition 3. 1. $f(z;0) = \frac{\lambda}{\lambda+g} f(z;1) + \begin{cases} \frac{g}{\lambda+g} z = 0 \\ 0 z \neq 0 \end{cases}$ so that $f(0;0) = \frac{\lambda}{\lambda+g} f(0;1) + \frac{g}{\lambda+g}$

2. If $x > 0$ That $f(z;x) = \sum_{n=1}^{\infty} (R(n,1) e^{-gn\rho} \sum_{k=0}^{\infty} [\frac{(gn\rho)^k}{k!} e^{-gn\rho}) f(z, x - n + k)]$ From condition (41) we obtain $f(0;0) = \frac{\lambda}{\lambda+g} f(0;1) + \frac{g}{\lambda+g}$ if $z \neq 0$ so $f(z;0) = \frac{\lambda}{\lambda+g} f(z;1)$.

Proof for condition 1. Let there be no cars in front of the traffic light at the moment the green light turns on: $n = 0$. Let the event $A(z;0)$ there is "at the moment the green signal lights up in front of the traffic light there are no cars, and by the time it is extinguished there are z cars in front of it." We are talking about finding the probability of this event, in other notation, about calculating $f(z;0)$. We put forward two hypotheses about the course of this process: H_1 = "First, a car will arrive at the traffic light, and then the green light will turn off." Proposition 1 implies that $P(H_1) = \frac{\lambda}{\lambda+g}$. At the same time, at any moment of the green light burning after the arrival of the car, we fix $x = 1$. Notice, that $P(A(z;0)/H_1) = f(z;1)$ by the property of the exponential distribution. Probability of the opposite event – H_2= "The car has not yet arrived, and the green light has already gone out" is equal to $\frac{g}{\lambda+g}$. At any time when the green signal is on, we fix $x = 0$

Moreover, it is clear that at this moment and $z = 0$ those. an event happened $A(0;0)$ so $P(A(0;0)/H_2) = 1$. Therefore, we have $f(0;0) = \frac{\lambda}{\lambda+g} f(0;1) + \frac{g}{g+\lambda}$ if $z \neq 0$ then this cannot be $P(A(z;0)/H_2) = 0$. According to the total probability formula in this case, from Proposition 3 we obtain: $f(0\,;0)=\frac{\lambda}{\lambda+g}f(0\,;1)+\frac{g}{\lambda+g}$ where we get $f(0;1)=\frac{\lambda+g}{\lambda}f(0;0)-\frac{g}{\lambda}$

Proof for condition 2. Calculate the probability $f(z;x)$ at $x>0$. While 1 car passes through the traffic light, during this time more arrive α_1 cars and stand in line. While first in line α_1 cars pass through the traffic light, more arrive α_2 cars, etc. Let during the journey α_j no more cars coming. Let $1+\alpha_1+\ldots+\alpha_j=n$. To these n cars managed to pass through the traffic light, the green signal should be on for at least $n\rho$ units of time, the probability of this $e^{-gn\rho}$. But the queue has shrunk n cars at the expense of their passage through the traffic light, so that it left $x-n$. Let this time besides $1,\alpha_1,\ldots,\alpha_j$ cars rigidly connected with the traffic light 1 car through the traffic light - more approached the traffic light $k\geq 0$ cars, total, thus, for the time $n\rho$ arrived k cars and standing in line $(x-n+k)$ cars. Probability this is equal to $\frac{(gn\rho)^k}{k!}e^{-gn\rho}$. So we get $f(z;x)=$ $\sum_{n=1}^{\infty}(R(n,1)e^{-gn\rho}\sum_{k=0}^{\infty}\frac{(gn\rho)^k}{k!}e^{-gn\rho}f(z;x-n+k)$. Denoting $f(z;l)$ through y_l, we obtain an infinite system of linear algebraic equations with an infinite number of unknowns y_l, which, in principle, can be solved by methods suitable for finite systems with a finite number of unknowns and find the quantities $y_l=f(z;l)$.

2.5. About stationary mode

A traffic light passes on average during the duration of the green light $A=\frac{1}{gT}$ cars, and for the average travel time cars through the traffic light arrive at it (also on average) $B=\lambda(\frac{1}{g}+\frac{1}{r})=\lambda$ cars.

Proposition 4. If $B>A$ i.e. if $\lambda gT>1$ then there is no steady state. This is obvious, because under this condition the traffic light simply cannot cope with the flow of arriving cars. If $B<A$ then, apparently, a stationary regime exists.

Estimated travel time. Let's consider one more case when the traffic light crossing time is random and distributed according to the exponential law: $P(t>c)=e^{-\mu c}$ where μ distribution parameter. Meaningful meaning of the parameter μ The average time to cross a traffic light is $\frac{1}{\mu}$. There are some grounds to consider the real time of moving to be approximately exponentially distributed. With such a distribution, most of the cars pass the traffic light in a relatively short time, and only a small part of the cars spends a relatively long time crossing.

With an exponential time of the crossing occupation, the passage of cars through the crossing can be modeled by a Markov process using a labeled graph built on a set of states, and all questions can be solved to the end. Let's denote the state "green light on, x cars waiting in line" by (z, x), and the state "red light on, x cars waiting in line" by (k, x), we also note that λ, g, r denote the parameters of the exponential distributions of the duration of the arrival interval between two cars and the burning times of the green and red lights. Through μ denote the parameter of the exponential time of passage through the traffic light. Thus, the probability that the time of the move t will exceed a is equal to $P(t>a)=e^{-\mu a}$. The labeled state graph has the following form (Fig. 2).

Now you can compose a system of linear algebraic equations as follows:

For each k -th state, we compose an equation $\sum_{i\neq k}\lambda_{ik}P_i=(\sum_{j\neq k}\lambda_{kj})\,P_k$ where λ_{ij} density of transition probability (corresponds to an arrow going from the i -th state to the j -th). To these equations we add one more - normalization: the sum of all probabilities is equal to 1: $\sum_i P_i=1$.

The solution of this system gives the probabilities of states in the stationary mode. We write the corresponding equation for the state (3, x + 1). For simplicity, we assign numerical values to the states -((3,x)-1, (3,x+1)-2,(3,x-1)-3,(K,x)-4,(K,x+1)-5)

$$\mu P_2+\lambda P_3+gP_4=(\lambda+\mu+r)P_1$$

The labeled state graph above, together with the normalization equation, allows us to find the limiting or stationary probabilities P_k. After finding these probabilities, you can answer questions, including those posed at the very beginning:

Average queue length $\sum_{x>0}[P(3,x)+P(K,x)$

Probability of passing without delay = $P(3{,}0)+P(K,0)$.

Average waiting time at the traffic light = Average length of the queue multiplied by the average

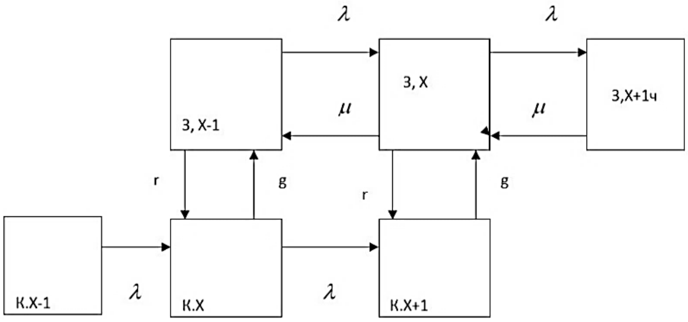

Fig. 2. Labeled state graph of the system "traffic flow - intersection"

time the crossing was occupied by one car, which is equal to $1/\mu$.

Other similar questions can be studied in a similar way.

3. Discussion and Conclusions

The presented analysis shows the possibility of using queuing theory formulas in evaluating the effectiveness of the functioning of the intelligent transport infrastructure, which makes it possible to increase the reliability of economic calculations when determining the costs of both renewal and design.

As a result of the study, an accurate mathematical description of the module's operation logic is presented, which, when integrated into the system of urban intelligent regulation of traffic flows, will allow moving from probabilistic models to an economic assessment of the efficiency of the transport infrastructure. The proposed method is a modification of the fundamental results of E. Borel and F. Haight related to the simplest flow of events, which, through additional restrictions, reduces the operation of the transport node to a Markov random process and this allows using the formulas of queuing theory when considering its functioningfor further economic assessment.

A labeled state graph has been built that allows obtaining quantitative characteristics of the operation of the transport node. The theoretical basis of computational experiments intended for solving applied problems in the future has been created.

At the same time, important conditions for the correct operation of the proposed method should be noted. The application of the method assumes a number of conditions - the Poisson flow of the transport arriving at the intersection, the indicative time for the transport to pass through the intersection, the random mode of the traffic light. The mode of allowing the movement of traffic lights has the property of lack of memory, that is, it is an indicative law. Consideration of the functioning of transport interchanges as a Markov random process makes it possible to use the mathematical apparatus of the queuing theory in full.It was possible for the first time to obtain an infinite system of linear algebraic equations with an infinite number of unknowns, which can later be solved by methods suitable for finite systems with a finite number of unknowns.

However, the above conditions must be met. According to the authors, the proposed method can be an alternative to the more costly method of agent-based modeling. A separate study will require the synchronization of the modified method with the fuzzy control logic of the regulation of traffic flows.

References

1. Ouallane, A.A., Bahnasse, A., Bakali, A., Talea, M. Overview of Road Traffic Management Solutions based on IoT and AI // Procedia Computer Science. Vol. 198, 2022. P. 518 − 523. https://doi.org/10.1016/j.procs.2021.12.279.
2. Shamlitskiy, Y., Popov, A., Saidov, N., Moiseeva, K. Transport Stream Optimization Based on Neural Network Learning Algorithms (XIII International Conference on Transport Infrastructure: Territory Development and Sustainability) // Transportation Research Procedia. Vol. 68, 2023. P. 417 − 425. https://doi.org/10.1016/j.trpro.2023.02.056.
3. Degrande, T., Vannieuwenborg, F., Verbrugge, S., Colle, D. Deployment of Cooperative Intelligent Transport System infrastructure along highways: A bottom-up societal benefit analysis for Flanders // Transport Policy. Vol. 134, April 2023. P. 94-105. https://doi.org/10.1016/j.tranpol.2023.02.013.
4. Korablev, V., Gugutishvili, D., Lepekhin A., Gerrits B. Developing a Traffic Management System Architecture Model // Transportation Research Procedia. Vol. 54, 2021. P. 918 − 926. https://doi.org/10.1016/j.trpro.2021.02.147.

5. Bhouri, N., Aron, M., Lebacque, J. P, Haj-Salem, H. Effectiveness of travel time reliability indicators in the light of the assessment of dynamic managed lane strategy // Journal of Intelligent Transportation Systems. Vol. 21, Issue 6, 2017. P. 492 – 506. https://doi.org/10.1080/15472450.2017.1327815
6. Wang, Z., Delahaye, D., Farges, J.-L., Alam S. Complexity optimal air traffic assignment in multi-layer transport network for Urban Air Mobility operations // Transportation Research Part C: Emerging Technologies. Vol. 142, September 2022, 103776. P. 1 – 23. https://doi.org/10.1016/j.trc.2022.103776
7. Gorodokin, V., Zhankaziev, S., Shepeleva, E., Magdin, K., Evtyukov, S. Optimization of adaptive traffic light control modes based on machine vision (International conference of Arctic transport accessibility: networks and systems) // Transportation Research Procedia. Vol. 57, 2021. P.241 – 249. https://doi.org/10.1016/j.trpro.2021.09.047
8. Glushkov, A., Shepelev V., Slobodin, I., Zhankaziev, S., Myachkov K. Construction of a System for Dynamic Monitoring of Pollutant Emissions by Transport Flows // Transportation Research Procedia. Vol. 68, 2023. P. 543 – 550. https://doi.org/10.1016/j.trpro.2023.02.074
9. Tahaei, H., Afifi, F., Asemi, F., Zaki, F., Anuar, N. B. The rise of traffic classification in IoT networks: A survey // Journal of Network and Computer Applications. Vol. 154, 15 March 2020, 102538. P. 1 – 20. https://doi.org/10.1016/j.jnca.2020.102538
10. Oumaima, E. J., Jalel, B. O., Veronique V. A stochastic mobility model for traffic forecasting in urban environments // Journal of Parallel and Distributed Computing. Vol. 165, July 2022. P. 142 – 155. https://doi.org/10.1016/j.jpdc.2022.03.005
11. Ibili, F., Owolabi A. O., Ackaah, W., Massaquoi, A. B. Statistical modelling for urban roads traffic noise levels // Scientific African. Vol. 15, March 2022, e01131. P. 1 – 9. https://doi.org/10.1016/j.sciaf.2022.e01131
12. Rosero, F., Fonseca, N., Mera, Z., López, J.-M. Assessing on-road emissions from urban buses in different traffic congestion scenarios by integrating real-world driving, traffic, and emissions data // Science of The Total Environment. Vol. 863, 10 March 2023, 161002. P. 1 – 17.
13. Harrou, F., Zeroual, A., Hittawe, M. M., Sun, Y. Chapter 2 – Road traffic modeling // Road Traffic Modeling and Management. Using Statistical Monitoring and Deep Learning. 2022. P. 15 – 63. https://doi.org/10.1016/B978-0-12-823432-7.00007-0
14. Guchenko, M., Sokhin, N., Skalsky, A., Bartuška, L., Čejka., J. Research of Prognostic Abilities of Local Model of Controlled Process for Traffic Forecasting // Transportation Research Procedia. Vol. 44, 2020. P. 78 – 85. https://doi.org/10.1016/j.trpro.2020.02.012
15. Glushkov, A., Shepelev, V., Vorobyev, F., Mavrin, A., Marusin, A., Evtykov, A. Analysis of the intersection throughput at changes in the traffic flow structure // Transportation Research Procedia. Vol. 57, 2021. P. 192 – 199. https://doi.org/10.1016/j.trpro.2021.09.042
16. Abbracciavento, F., Zinnari, F., Formentin, S., Bianchessi, A. G., Savaresi, S. M. Multi-intersection traffic signal control: A decentralized MPC-based approach // IFAC Journal of Systems and Control. Vol. 23, March 2023, 100214. P. 1 – 10. https://doi.org/10.1016/j.ifacsc.2022.100214
17. Rudskoy, A., Ilin, I., Prokhorov, F. Digital Twins in the Intelligent Transport Systems // Transportation Research Procedia. Vol.54, 2021. P. 927 – 935. https://doi.org/10.1016/j.trpro.2021.02.152
18. Castaneda, K., Sanchez, O., Herrera, R. F., Pellicer, E., Porras H. BIM-based traffic analysis and simulation at road intersection design // Automation in Construction. Vol. 131, November 2021, 103911. P. 1 – 14. https://doi.org/10.1016/j.autcon.2021.103911
19. Lu, X., Wang, H. Z. J., Zhang, M., Zhou, C., Zhang, H. Modeling impacts of the tunnel section on the mixed traffic flow: A case study of Jiaodong'ao Tunnel in China // Physica A: Statistical Mechanics and its Applications. Vol. 603, 1 October 2022, 127840. P. 1 – 18. https://doi.org/10.1016/j.physa.2022.127840
20. Mfenjou, M. L., Ari, A. A. A., Abdou, W., Spies, F., Kolyang. Methodology and trends for an intelligent transport system in developing countries // Sustainable Computing: Informatics and Systems. Vol. 19, September 2018. P. 96 – 111. https://doi.org/10.1016/j.suscom.2018.08.002
21. Jena, S., Patro S., Dutta M., Bhuyan, P.K. Assessment of service quality at unsignalized intersections using traffic simulation and computational intelligence // Transportation Letters. Vol. 14, Iss. 4, May 2022, P. 365 – 377. https://doi.org/10.1080/19427867.2020.1868179
22. Osipov, A.; Pleshakova, E.; Gataullin, S.; Korchagin, S.; Ivanov, M.; Finogeev, A.; Yadav, V. Deep Learning Method for Recognition and Classification of Images from Video Recorders in Difficult Weather Conditions. Sustainability 2022, 14, 2420. https://doi.org/10.3390/su14042420
23. Spirin, I.V. Organization and management of passenger road transport. In Transportation Organization and Transport Management; Academy: Moscow, Russia, 2005; p. 396.
24. Borel E., Sur l'emploi du Théorème de Bernoulli pour Faciliter le Calcul d'un Infinité de Coefficients, Application au Probleme de L' attente a un Guichet, Compt. rend. acad. sci. Paris, 214, 452-456 (1942)
25. Saaty, Thomas L. Elements of queueing theory: with applications. Vol. 34203. New York: McGraw-Hill, 1961.
26. Haight, Frank A. Mathematical theories of traffic flow. 1965.
27. Ivanyuk, V. Forecasting of digital financial crimes in Russia based on machine learning methods. J Comput Virol Hack Tech (2023). https://doi.org/10.1007/s11416-023-00480-3
28. Stack Exchange Network [Electronic resource]:Mathematics\\ https://math.stackexchange.com/questions/1332413/comparing-two-exponential-random- variables(accessdate 07/31/2023).

Gataullin Timur G. Deputy Director of the Center for Digital Economy, Professor of the Department of Mathematical Methods in Economics and Management. He is a member of the editorial board of the journal "Digital Economy" and CEMI RAS.Research interests: mathematical modeling in economics, knowledge economy. E-mail:t.m.gataullin@mtuci.ru

Osipov Alexey V. MIREA - Russian Technological University, Moscow, Russia, Moscow, Russian Federation. Candidate of Physical and Mathematical Sciences. Research interests: information security, artificial intelligence, machine learning. E-mail: a.v.osipov@mtuci.ru

Pleshakova Ekaterina S. MIREA - Russian Technological University, Moscow, Russia, Moscow, Russian Federation. Candidate of Technical Sciences. Research interests: information security, machine learning. E-mail: espleshakova@fa.ru

Bylevskiy Pavel G. Candidate of Philosophy, Associate Professor, Moscow State Linguistic University.Research interests: information security.E-mail: pr-911@yandex.ru

Gataullin Sergey T. MIREA - Russian Technological University, Moscow, Russia, Moscow, Russian Federation. Research interests: mathematical methods of decision making, economic and mathematical modeling, information security, machine learning, artificial intelligence. E-mail: s.t.gataullin@mtuci.ru

Математическое моделирование для развития интеллектуальной транспортной инфраструктуры

Т. М. Гатауллин[I], Е. С. Плешакова[III], П. Г. Былевский[II], А. В. Осипов[III], С. Т. Гатауллин[III,IV]

[I] Новая экономическая ассоциация, Москва, Россия
[II] Московский государственный лингвистический университет, Москва, Россия
[III] МИРЭА - Российский технологический университет, Москва, Россия
[IV] Центральный экономико-математический институт РАН, Москва, Россия

Аннотация: Для повышения достоверности технико-экономических обоснований развития интеллектуальной транспортной инфраструктуры в условиях устойчивого развития мегаполисов необходимо более тщательно учитывать фактические условия эксплуатации одноуровневых пересечений транспортных потоков. Решение этой задачи требует использования передовых экономико-математических инструментов. Математическое описание представляет собой модификацию фундаментальных результатов Э. Бореля и Ф. Хейта, относящихся к простейшему течению событий. Применение этого метода позволит как оценивать экономическую целесообразность расширения существующей транспортной инфраструктуры, так и проектировать новые транспортные объекты. Данный подход может быть применен на этапе первоначальной количественной оценки целесообразности замены одноуровневой развязки транспортных потоков многоуровневой развязкой во избежание экономических потерь.

Ключевые слова: математическое моделирование, устойчивое развитие, цифровая экономика, интеллектуальная транспортная инфраструктура, теория массового обслуживания.

DOI 10.14357/20718632240308 **EDN** OYISFY

Литература

1. Ouallane, A.A., Bahnasse, A., Bakali, A., Talea, M. Overview of Road Traffic Management Solutions based on IoT and AI // Procedia Computer Science. Vol. 198, 2022. P. 518 − 523. https://doi.org/10.1016/j.procs.2021.12.279.
2. Shamlitskiy, Y., Popov, A., Saidov, N., Moiseeva, K. Transport Stream Optimization Based on Neural Network Learning Algorithms (XIII International Conference on Transport Infrastructure: Territory Development and Sustainability) // Transportation Research Procedia. Vol. 68, 2023. P. 417 − 425. https://doi.org/10.1016/j.trpro.2023.02.056.
3. Degrande, T., Vannieuwenborg, F., Verbrugge, S., Colle, D. Deployment of Cooperative Intelligent Transport System infrastructure along highways: A bottom-up societal benefit analysis for Flanders // Transport Policy. Vol. 134, April 2023. P. 94-105. https://doi.org/10.1016/j.tranpol.2023.02.013
4. Korablev, V., Gugutishvili, D., Lepekhin A., Gerrits B. Developing a Traffic Management System Architecture Model // Transportation Research Procedia. Vol. 54, 2021. P. 918 − 926. https://doi.org/10.1016/j.trpro.2021.02.147
5. Bhouri, N., Aron, M., Lebacque, J. P, Haj-Salem, H. Effectiveness of travel time reliability indicators in the light of the assessment of dynamic managed lane strategy // Journal of Intelligent Transportation Systems. Vol. 21, Issue 6, 2017. P. 492 − 506. https://doi.org/10.1080/15472450.2017.1327815.
6. Wang, Z., Delahaye, D., Farges, J.-L., Alam S. Complexity optimal air traffic assignment in multi-layer transport network for

Urban Air Mobility operations // Transportation Research Part C: Emerging Technologies. Vol. 142, September 2022, 103776. P. 1 − 23. https://doi.org/10.1016/j.trc.2022.103776.
7. Gorodokin, V., Zhankaziev, S., Shepeleva, E., Magdin, K., Evtyukov, S. Optimization of adaptive traffic light control modes based on machine vision (International conference of Arctic transport accessibility: networks and systems) // Transportation Research Procedia. Vol. 57, 2021. P.241 − 249. https://doi.org/10.1016/j.trpro.2021.09.047.
8. Glushkov, A., Shepelev V., Slobodin, I., Zhankaziev, S., Myachkov K. Construction of a System for Dynamic Monitoring of Pollutant Emissions by Transport Flows // Transportation Research Procedia. Vol. 68, 2023. P. 543 − 550. https://doi.org/10.1016/j.trpro.2023.02.074.
9. Tahaei, H., Afifi, F., Asemi, F., Zaki, F., Anuar, N. B. The rise of traffic classification in IoT networks: A survey // Journal of Network and Computer Applications. Vol. 154, 15 March 2020, 102538. P. 1 − 20. https://doi.org/10.1016/j.jnca.2020.102538.
10. Oumaima, E. J., Jalel, B. O., Veronique V. A stochastic mobility model for traffic forecasting in urban environments // Journal of Parallel and Distributed Computing. Vol. 165, July 2022. P. 142 − 155. https://doi.org/10.1016/j.jpdc.2022.03.005.
11. Ibili, F., Owolabi A. O., Ackaah, W., Massaquoi, A. B. Statistical modelling for urban roads traffic noise levels // Scientific African. Vol. 15, March 2022, e01131. P. 1 − 9. https://doi.org/10.1016/j.sciaf.2022.e01131.
12. Rosero, F., Fonseca, N., Mera, Z., López, J.-M. Assessing on-road emissions from urban buses in different traffic congestion scenarios by integrating real-world driving, traffic, and emissions data // Science of The Total Environment. Vol. 863, 10 March 2023, 161002. P. 1 − 17.
13. Harrou, F., Zeroual, A., Hittawe, M. M., Sun, Y. Chapter 2 − Road traffic modeling // Road Traffic Modeling and Management. Using Statistical Monitoring and Deep Learning. 2022. P. 15 − 63. https://doi.org/10.1016/B978-0-12-823432-7.00007-0.
14. Guchenko, M., Sokhin, N., Skalsky, A., Bartuška, L., Čejka., J. Research of Prognostic Abilities of Local Model of Controlled Process for Traffic Forecasting // Transportation Research Procedia. Vol. 44, 2020. P. 78 − 85. https://doi.org/10.1016/j.trpro.2020.02.012.
15. Glushkov, A., Shepelev, V., Vorobyev, F., Mavrin, A., Marusin, A., Evtykov, A. Analysis of the intersection throughput at changes in the traffic flow structure // Transportation Research Procedia. Vol. 57, 2021. P. 192 − 199. https://doi.org/10.1016/j.trpro.2021.09.042.
16. Abbracciavento, F., Zinnari, F., Formentin, S., Bianchessi, A. G., Savaresi, S. M. Multi-intersection traffic signal control: A decentralized MPC-based approach // IFAC Journal of Systems and Control. Vol. 23, March 2023, 100214. P. 1 − 10. https://doi.org/10.1016/j.ifacsc.2022.100214.
17. Rudskoy, A., Ilin, I., Prokhorov, F. Digital Twins in the Intelligent Transport Systems // Transportation Research Procedia. Vol.54, 2021. P. 927 − 935. https://doi.org/10.1016/j.trpro.2021.02.152.
18. Castaneda, K., Sanchez, O., Herrera, R. F., Pellicer, E., Porras H. BIM-based traffic analysis and simulation at road intersection design // Automation in Construction. Vol. 131, November 2021, 103911. P. 1 − 14. https://doi.org/10.1016/j.autcon.2021.103911.
19. Lu, X., Wang, H. Z. J., Zhang, M., Zhou, C., Zhang, H. Modeling impacts of the tunnel section on the mixed traffic flow: A case study of Jiaodong'ao Tunnel in China // Physica A: Statistical Mechanics and its Applications. Vol. 603, 1 October 2022, 127840. P. 1 − 18. https://doi.org/10.1016/j.physa.2022.127840.
20. Mfenjou, M. L., Ari, A. A. A., Abdou, W., Spies, F., Kolyang. Methodology and trends for an intelligent transport system in developing countries // Sustainable Computing: Informatics and Systems. Vol. 19, September 2018. P. 96 − 111. https://doi.org/10.1016/j.suscom.2018.08.002.
21. Jena, S., Patro S., Dutta M., Bhuyan, P.K. Assessment of service quality at unsignalized intersections using traffic simulation and computational intelligence // Transportation Letters. Vol. 14, Iss. 4, May 2022, P. 365 − 377. https://doi.org/10.1080/19427867.2020.1868179.
22. Osipov, A.; Pleshakova, E.; Gataullin, S.; Korchagin, S.; Ivanov, M.; Finogeev, A.; Yadav, V. Deep Learning Method for Recognition and Classification of Images from Video Recorders in Difficult Weather Conditions. Sustainability 2022, 14, 2420. https://doi.org/10.3390/su14042420.
23. Spirin, I.V. Organization and management of passenger road transport. In Transportation Organization and Transport Management; Academy: Moscow, Russia, 2005; p. 396.
24. Borel E., Sur l'emploi du Théorème de Bernoulli pour Faciliter le Calcul d'un Infinité de Coefficients, Application au Probleme de L' attente a un Guichet, Compt. rend. acad. sci. Paris, 214, 452-456 (1942)
25. Saaty, Thomas L. Elements of queueing theory: with applications. Vol. 34203. New York: McGraw-Hill, 1961.
26. Haight, Frank A. Mathematical theories of traffic flow. 1965.
27. Ivanyuk, V. Forecasting of digital financial crimes in Russia based on machine learning methods. J Comput Virol Hack Tech (2023). https://doi.org/10.1007/s11416-023-00480-3.
28. Stack Exchange Network [Electronic resource]:Mathematics\\ https://math.stackexchange.com/questions/1332413/comparing-two-exponential-random- variables(accessdate 07/31/2023).

Тимур Малютович Гатауллин. Московский технический университет связи и информатики, Москва, Россия. Заместитель директора, профессор. Область научных интересов: математическое моделирование в экономике, экономика знаний. E-mail: t.m.gataullin@mtuci.ru

Алексей Викторович Осипов. МИРЭА - Российский технологический университет, Москва, Россия. Доцент. Кандидат физико-математических наук. Область научных интересов: информационная безопасность, искусственный интеллект, машинное обучение. E-mail: a.v.osipov@mtuci.ru

Екатерина Сергеевна Плешакова. МИРЭА - Российский технологический университет, Москва, Россия. Доцент. Кандидат технических наук. Область научных интересов: информационная безопасность, машинное обучение. E-mail: espleshakova@fa.ru

Павел Геннадиевич Былевский. Московский государственный лингвистический университет, Москва, Россия. Доцент. Кандидат философских наук, доцент. Область научных интересов: информационная безопасность. E-mail: pr-911@yandex.ru

Сергей Тимурович Гатауллин. МИРЭА - Российский технологический университет, Москва, Россия. Кандидат экономических наук. Область научных интересов: математические методы принятия решений, экономико-математическое моделирование, информационная безопасность, машинное обучение, искусственный интеллект. E-mail: s.t.gataullin@mtuci.ru